ABSTRACTS

SECTION A
EDUCATION

ВОВЕД ВО ИТЕРАЦИЈА НА ФУНКЦИИ И ЈУЛИА МНОЖЕСТВАТА

Roza Aceska

АПСТРАКТ. Итерациите на полиноми и рационални функции се интензивно проучувани во почетокот на 20-от век од Гастон Јулиа. При итерирање на некоја функција може да се добијат различни резултати. Преку примерот на итерирање на реална/комлексна квадратна функција учениците се запознаваат со различните исходи на итерирањето. Со користење на компјутерска програма се добиваат интересни графици чии ефекти се поврзани со брзините на итерирање. Во ова излагање даден е пример, лесен за пресметки (со обичен дигитрон) и за разбирање на однесувањето на итератите (ограничени или не). Областа е интересна за работа со надарени ученици.

SYSTEMS OF LINEAR EQUATIONS

Vladimir Baltic

ABSTRACT. Systems of linear equations are common in science and mathematics. They find their application in physics, chemistry, economy, linear programming and in plenty of other scientific areas. We will present a brief survey of methods for solving the system: Gauss' method, Gauss-Jordan reduction, Cramer's Rule, graph method. Furthermore, we will give remarks on important things in every method. Additionally, we will introduce an application of the linear independent row-vectors in the elementary teaching of solving the systems of linear equations.

ПРИМЕНА НА НЕКОИ ОД ТЕХНИКИТЕ ОД ПРОЕКТОТ ЧПКМ - СОРОС ВО НАСТАВАТА ПО МАТЕМАТИКА

Sofija Brezovska, Gorgi Kitanski, Jelena Babovic, Anastazija Belevska, Biljana Rejcevic, Biljana Jovanceva

АПСТРАКТ. Поради потребата за осовременување на наставата сакаме да предложиме некои техники за подобро совладување на материјалот по математика. Техники кои ќе придонесат за креативно осмислување и реализирање на

ХАРМОНИСКИ ПРИНЦИП ВО МОДЕЛИТЕ НА РАСТЕЊЕ НА ПОПУЛАЦИЈА

Dragica Radovanovic

АПСТРАКТ. Матеметиката отсекогаш имала многу важна улога во биологијата и другите природни накуки. Во овој труд се прикажани основните математички модели на популациската биологија - област за чие втемелување (50-тите и 60-тите години на дваесетиот век) заслужни се Роберт Мекартур и Едвард Вилсон, кои почувствувале дека за опишување на природните закони кои управуваат со растењето на популација наместо едноставни текстуални објаснувања може да се користат математички модели. После многубројни експерименти нивниот заклучок бил - *Во природата постои прекрасен стремеж нештата и појавите да се урамнотежат и да се достигне хармонија и кога би ја оставиле природата сама да се уредува би постоел еквилибриум.*

Еден од најпознатите модели на растење на популација е логистичкиот модел на растење кој е претставен со доста едноставна равенка, меѓутоа веќе тука можме да дојдеме до многу интересни заклучоци - во зависност од стапката на растење на популацијата бројот на единки на популацијата монотоно или со пригушени осцилации го достигнува еквилибриумот, или осцилира меѓу неколку фиксни вредности, но може да се однесува и хаотично.

STEINER SYSTEMS AND SYMMETRICAL DESIGN APPLICATION IN THE AGRICULTURAL EXPERIMENT ORGANIZATION

D. M. Randjelovic, M. D. Randjelovic, J. Jovanovic, M. Jašovic

ABSTRACT. Presentation, construction and conditions of existence one combinatorial configuration–design is in the base of determining the possibility some experiment organization in many science discipline and also in agriculture. With the design B(v,r_1,...,r_v,b,k_1,...,k_b,l{12},...,l{v-1,v}), like one combinatorial configuration, can be presented organization one's experiment in which participate finite number v elements some basic sets, which should organise in the b designs from defined number k_b elements from this basic sets, but so that every of this elements are exactly in r_v designs and every pair of this different elements is in l{v-1,v} designs. In this paper are considered the conditions of presentation, construction and also existence one "narrower" class of designs so-called balanced

incomplete block designs(BIBDs)-Steiner systems which are balanced incomplete block-designs with l{v-1,v}=1. Also is cosidered and the class of symmetrical designs which are BIBDs with b=v(and automatic k=r) - B(v,k,l) designn. Three examples of Steiner system and symmetrical designs application in agricultural experiment organization is also given in the paper.

LANCHESTER COMBAT ATTRITION DIFFERENTIAL MODELS

Nevena Serafimova

ABSTRACT. Lanchester-type differential equation attrition models refer to the set of models that describe over time changes in the force levels of combatants and other significant variables that describe the combat process. Their solution can provide an insight into the dynamics of combat, and could be applied almost through the whole hierarchical system of combat activities. This introductory work looks into different types of these models, their solutions and the possibilities for their utilization and upgrading.

АПРОКСИМАЦИЈА НА ЕМПИРИСКИТЕ КОН НАЈПОЗНАТИТЕ ТЕОРИСКИ ДИСТРИБУЦИИ

Kosta Sotiroski

АПСТРАКТ. Спознавањето на законитостите на економските појави и процеси е неопходно во процесот на економските истражувања. Тоа е основа за идни предвидувања, прогнозирања, проекции и симулација на економските појави и процеси. За таа цел примената на теориските дистрибуции, со соодветна софтверска подршка, во процесот на апроксимација е во функција на добивање целосна, потполна и научно заснована информација за тековите и развојот на економските појави и процеси.

КЛУЧНИ ЗБОРОВИ. теориски дистрибуции, статситички калкулатори и пакети, ссстем за подршка на одлучувањето.

CITIES IN-BETWEEN CHAOS AND HARMONY

Jasna Stefanovska

ABSTRACT. Cities exist more than 5500 years, and till now, it is impossible to give a single definition what city is, but it is absolutely true that large numbers of people living in close proximity do not in themselves constitute a city. Cities are too complicated, too far beyond our control, and affect too many people, who are subject to too many cultural variations, to permit any rational answer.

Steiner systems and symmetrical design application in the agricultural experiment organization

D.M.Randjelović, M.D.Randjelović, J.Jovanović, M.Jašović

Presentation, construction and conditions of existence one combinatorial configuration - design is in the base of determining the possibility some experiment organization in many science discipline also in agriculture.With the design $B(v, r_1, ..., r_v, b, k_1, ..., k_b, \lambda_{12}, ..., \lambda_{v-1,v})$ like one combinatorial configuration, can be presented organization one's experiment in which participate finite number v elements some basic sets, which should organise in the b designs from defined number k_b elements from this basic sets, but so that every of this elements are exactly in r_v designs and every pair of this different elements is in $\lambda_{v-1,v}$ designs. In this paper are considered the conditions of presentation, construction and also existence one "narrower" class of designs so-called balanced incomplete block designs(BIBDs) -Steiner systems which are balanced incomplete block-designs with $\lambda_{v-1,v}= 1$. Also is cosidered and the class of symmetrical designs which are BIBDs with b=v and automatic k=r - $B(v, k, \lambda)$ designn. Three examples of Steiner system and symmetrical designs applicationin in agricultural experiment organization is also given in the paper.

Key Words: Steiner system, symmetrical design, agricultural experiment organization

1. Presentation of BIBDs

Definition 1.1 Suppose that M is finite or infinite set. Every set of subsets, which consists elements of set M, is called configuration over the set M, it is marked with J and it is represented in form $J = \{S_1, S_2, ..., S_m\}$ where every subset S_1 contains random number of elements.
Configuration can be graphically presented so that we assign points of plane to the elements of the set $M = \{x_1, x_2, ..., x_n\}$ and every point, which belongs to set S_i , must be circled with a curve line.
The configuration can be given with a graph where the points of plane (like elements of set M) are connected with appropriate circle in planes that represent subsets S_i .

Because of elaborated matrix-calculation the configuration is represented by so-called: incident matrix. Suppose that $J = \{S_1, S_2, ..., S_m\}$ is configuration over the set $M = \{x_1, x_2, ..., x_n\}$. For element x_j , j=1,2,...,n, we say that it is in incidence with subset S_i i=1,2,...,m, if $x_j \in S_i$. Rectangular (0,1)-matrix $A = \{a_{ij}\}$, in form [m x n], whose elements are for each i = 1,2,...,m, and for each j =1,2,...,n defined with

$$a_{ij} = \begin{cases} 1, & \text{if } x_j \in S_i \\ 0, & \text{if } x_j \neq S_i, \end{cases}$$

It is called incident matrix of a given configuration J over the set M.

Because the configuration over the set is given by subsets and because elements in those subsets are not organized and because schedule of this subsets in the configuration is not important, we can say that different notices of elements in basic set, and also subsets in configuration are possible.
It leads us to the conclusion that one configuration can be corresponded by more incident matrix. The question is: in which form we should give the configuration? The answer is very clear - we should change the configuration so that we get trivial form of incident matrix.

Definition 1.2 By design we mean any configuration $B = \{B_1, ..., B_b\}$ over finite set $V = \{a_1, a_2, ..., a_v\}$, where b i v are natural numbers. Design can be defined like definite pair (V,B) where $V = \{a_1, a_2, ..., a_v\}$ is finite set of elements, and $B = \{B_1, B_2, ..., B_b\}$ set of subsets of different elements from V, or we can say set of block's(we mean that $B_i \neq B_j$ for $i \neq j$).

Let us have design (V,B). For element a_j , $a_j \in V$ $j = 1, 2, ..., v$ we can say that it is incident to block B_i, $Bi \in B$ $i = 1, 2, ..., b$ if $a_j \in B_i$. With k_j, $j = 1, 2, ..., b$ we will mark total number of elements a_i , $a_i \in V$ $i = 1, 2, ..., v$ which is incident to block B_j . Total number of blocks B_j, $j = 1, 2, ..., b$, incident to element a_i, $i = 1, 2, ..., v$, we will mark with r_i . With λ_{it} we will mark the total number of elements of the set B_j (that kind of $a_i, a_t \in B_j$ forall $i = 1, 2, ..., v$ and $t = 1, 2, ..., v$, $i \neq j$). Because of undefinity of the elements of the blocks we can say that $\lambda_{it} = \lambda_{ti}$, so it is valid only to observe cases $i < t$. The numbers v, b, r_i, k_j, λ_{it}, are called arguments of given design.
The use of this kind of configuration in solving the combinatory problems is very complicated. That is the way in which we will observe only balanced incompleted block-designs marked with (v, r, b, k, λ)- configurations over finite set V where the set V consists of v mutual different elements and configuration B is made of b blocks, every block is made of exactly $k < v$ elements from V, every element from V appears in exactly $r < b$ blocks and every pair of different elements from V appears in exactly λ blocks.

2. Conditions for existence of designs

It can be proved that if one (v,r,b,k,λ)-configuration over the finite set of elements exists, than two arguments are in the roundly servitude of permanent three, what will be explained in the following theorem which will be given without proof.

Theorem 2.1[5] *If exists balanced design over finite set of elements V with arguments v,r,b,k,λ i.e exists (v,r,b,k,λ)- configuration over determinate set V, then the next equalities are correct:*

$$bk = vr \tag{2.1}$$

and

$$r \cdot (k-1) = \lambda \cdot (v-1) \ . \tag{2.2}$$

Theorem 2.1 gives necessary but not enough conditions for existence of designs. Namely, if some of arguments v,r,b,k,λ satisfy relations from theorem 2.1, we are not sure that suitable configuration really exists; also, since the arguments are natural numbers, by giving some three, sometimes it is not possible to define permanent two only by using relations from theorem 2.1. The large number of testing in existence of designs has been performed, thanks to the evolution of the computers, depending on some arguments that could satisfy conditions from theorem 2.1 it is decided :

- for large values of argument v, balanced incompleted block design always exists and for small values it never does.
- for k = 3 and k = 4 theorem gives enough conditions for appropriate configuration, but not for k = 5.

Let us have some (v,r,b,k,λ)-configuration over finite set of elements V and we know its incident matrix $A = \{aij\}$, which is rectangular, in form [b x v]. Every hers column contains r units, and every hers rows contains k units. Scalar product of two mutual different vector-columns is equal to the number of appearance of the pair of different elements from V in configuration, namely equals λ. The scalar product of any vector-column with himself equals argument r, namely equals the number of appearance of any elements V in design. Those obvious attributes of incident matrix of (v,r,b,k,λ)-configuration, enable us to get necessary and enough conditions of its existents.

Theorem 2.2[4] *Let the rectangular (0,1) matrix $A = \{aij\}$, in form [b x v] is incident matrix of some (v,r,b,k,λ)-configuration over finite set of elements V. Then the next equalities are correct:*

$$A^T \cdot A \ = \ (r-\lambda) \cdot I_v \ + \ \lambda \cdot J_v \ , \tag{2.3}$$

and

$$A \cdot J_{v \times 1} = k \cdot J_{b \times 1} \ .$$

Vice versa is also correct.

Unfortunately, for determining of the arguments using the theorem 2.2, it is necessary to solve suitable matrix equality and that is not possible without corresponding mathematics method which is until now not developed. Therefore we must take interest for cpecial designs. The most famous one is Steiners system for $\lambda = 1$. Interesting class, known as Steiner triple system, consists of class (v,r,b,k,1)-configuration for k=3.

Theorem 2.3[2] *Necessary and enough condition for the existence Steiner triple system are that v can be given in one of two form :*

$$v = 6t+1 \quad or \quad v = 6t+3 \quad for \quad t = 0,1,\ldots .$$

Theorem 2.4[1] *Steiner triple system exists if and only if for the argument v to be valid next equalities:*

$$v = 1(mod6) \quad or \quad v = 3(mod6) . \qquad (2.4)$$

In literature are known and Steiner quadruple system which are obtained for k=4 and $\lambda = 1$ i.e. $\lambda = 3$. Necessary and sufficient condition for theirs existence is given that must be satisfy one of the next two equations:

$$\begin{cases} \text{for } \lambda = 1: & v = 1(mod12) \text{ or } v = 4(mod12) \\ \text{for } \lambda = 3: & v = 0(mod4) \text{ or } v = 1(mod4) \end{cases} \qquad (2.5)$$

Definition 2.1 BIBDs with b=v (automatic follow k=r - see (2.1)) is symmetrical design.

Theorem 2.5[3] *If exists one $(v,k.\lambda)$ then :*
a)if v is even then is $k-\lambda)$ full square some natural number;
b)if v is odd then equation $z^2 = (k-\lambda)x^2 + (-1)^{(v-1)/2}\lambda y^2$ have in the set of natural numbers untrivial solutions for x,y,z.

Let us have some (v,r,b,k,λ)-configuration and b is divisible with r $(b \equiv 0(modr))$. Suppose that the blocks B_i for this configuration, $B = \{B_1, \ldots, B_b\}$, can be classified in r family $\beta_1, \beta_2, \ldots, \beta_r$, and in spite of that are two condition satisfied :

- The blocks from each family β_i, i=1,2,...,r , in the union consist all elements of V exactly once. (3.1)
- All the blocks which belong to one of family β_i, i=1,2,...,r are mutual disjunctive. (3.2)

Definition 2.2 (v,r,b,k,1)-configuration in which blocks B_i, i=1,2,...,b, can be grouped in r families and conditions (3.1) i (3.2) are satisfied is named solvable.

Remark 2.1 Condition $b \equiv 0(modr)$, which is valid for solvable design, is causing condition $v \equiv 0(modk)$.

Theorem 2.6[6] *Each $(n^2, n+1, n^2+n, n, 1)$-configuration , where n is a natural nummber, is solvable.*

3. Main results

The third problem in relation with BIBDs is theirs construction (presentation and existence are already considered).

Definition 3.1 Let we have matrix A like one incident matrix some (v,k,λ) - symmetrical configuration. The technique of receiving a new so called "dual design" using A^T is practicly based on the change of role the blocks and elements in starting matrix A.

Definition 3.2 Let we have some (v,k,λ) - symmetrical configuration over the set V, which is defined with the blocks $B_1, ..., B_v$ .We choose arbitrary B_v and form the new blocks $B_i' = B_i \backslash B_v$for $i = 1,2,...,v-1$. Now with the blocks B_i', ifor $i = 1,2,...,v-1$ we obtain a new so called "residual design" in relation with starting (v,k,λ) - symmetrical configuration.It is over the set $V' = V\backslash B_v$ with follow parameters v'=v-k,b'=v-1,k'=k-λ , $\lambda' = \lambda$.

Theorem 3.1[3] *Let natural numbers v,k,λ satisfy equality* $k(k-1) = \lambda(v-1)$ *for* $\lambda = 1,2$ *. Then if exist* $(v-k,k,v-1,k-\lambda,\lambda)$ *configuration exist and* (v,k,λ) *configuration which is residual in relation with starting design.*

The scientists make experiments to affirm theirs hypothesies. Instruments for that are the statistics methods but it is necessary before the beginning of one experiment to check its organization possibility. In agricultural experiments organization it is especially important wail they can be repeated often only next year. Mathematical instruments for that checking are on the basis of balaced incomplete block designs(BIBDs) while they can represent many agricultural experiments in which participate finite number different elements(sorts of herbs or races of cattle) which should organise in some blocks of defined number so that every of this elements are in other smaller number blocks and just every pair of this elements is in one of blocks.

Example 3.1 Let us have 6 new sorts of corn and they must be planted on 10 fields each divided on 3 lots; each of sorts is planted on 5 different fields so that each pair of sorts is planted on different field exactly once (twice). It is visible that the possibility of experiment organization depends of existence one balanced incomplete block design with argument v=6,r=5,b=10,k=3, $\lambda = 1(2)$ i.e. 6,5,10,3,1(2)-configuration over set V=1,2,3,4,5,6. Mark the sorts of corn with the number 1,2,3,4,5,6,the fields with B1,B2,B3,B4,B5,B6,B7,B8,B9,B10. We conclude that the existing of this configuration is possible only for $\lambda = 2$ with using the following known equality :
from (2.1) bk=vr => 10*3=6*5=30
and

from (2.2) r(k-1)= λ *((v-1) => 5*(3-1)=2*(6-1)=10.

So,for $\lambda = 2$ we can plant the sorts of corn on the next schedule:
B1=1,2,3,B2=1,2,5,B3=1,3,4,B4=1,4,6,B5=1,5,6
B6=2,3,6,B7=2,4,5,B8=2,4,6,B9=3,4,5,B10=3,5,6.

Example 3.2 Let's question if the organization of the following experiment is possible: let us have 15 new sorts of fruit and they must have pesticide tretman each day in the week; the tretman of pesticides is in groups of 3 sorts of fruit but so that after seven days each of sort is treated in group with remaining, exactly once. The possibility of experiment organization depends of existence 15,7,35,3,1-configuration over set V=1,2,3,4,5,6,7,8,9,10,11,12,13,14,15.
The blocks of Steiner triple system are:
B1=1,8,15,B2=2,3,5,B3=4,10,13,B4=6,9,14,B5=7,11,12
B6=2,9,15,B7=3,4,6,B8=5,11,14,B9=7,8,10,B10=1,12,13,
B11=3,10,15,B12=4,5,7,B13=6,8,12,B14=1,9,11,B15=2,13,14,
B16=4,11,15,B17=1,5,6,B18=7,9,13,B19=2,10,12,B20=3,8,14,
B21=5,12,15,B22=2,6,7,B23=1,10,14,B24=3,11,13,B25=4,8,9,
B26=6,13,15,B27=1,3,7,B28=2,8,11,B29=4,12,14,B30=5,9,10,
B31=7,14,15,B32=1,2,4,B33=3,9,12,B34=5,8,13,B35=6,10,11.
We conclude that the existing of this configuration is possible with using the following known equality :
from (2.1) bk=vr => b=v(v-1)/6 => 35=15*14/6=35
and
from (2.2) r(k-1)= ((v-1) => r=(v-1)/2 => 7=(15-1)/2=7.
Because conditions, necessary and enough,from Theorem 2.3 which are fulfilled:
from (2.4) v = 3(mod6) => 15=3(mod6),
the solution for weekly schedule of pesticide treatment of fruit is:
β_1=B1,B2,B3,B4,B5, β_2=B6,B7,B8,B9,B10, β_3=B11,B12,B13,B14,B15,
β_4=B16,B17,B18,B19,B20, β_5=B21,B22,B23,B24,B25,
β_6=B26,B27,B28,B29,B30, β_7=B31,B32,B33,B34,B35.

Example 3.3 Let we examine the existence of one (36,7,42,6,1) BIBD, using technique of residual BIBDs construction.
(36,7,42,6,1) BIBD satisfy the necessary condition for existence given with (2.1) and (2.2). But symmetrical configuration, if exist, must have next parameters: $v' = b' = b + 1 = 43$, $r' = k' = k + 1 = 7$, $\lambda' = \lambda = 1$, consequently (43,7,1)-configuration. On the basis of Theorem 2.5 equation $z^2 = 6x^2 - y^2$ must have untrivial solution for x,y and z and that is unpossible. So (43,7,1) symmetrical configuration, for which is given starting "residual design", can't exist and because of that and given starting (36,7,42,6,1) design can't exist.

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