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# AGGREGATION STATISTICS AND THE METHODS OF OPERATIONAL RESEARCH FOR WEIGHTING CRITERIA IN MULTIPLE CRITERIA DECISION MAKING

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**Key words:** Weights of criteria, DEA, AHP, Regression analysis, Factor analysis, Aggregated measure



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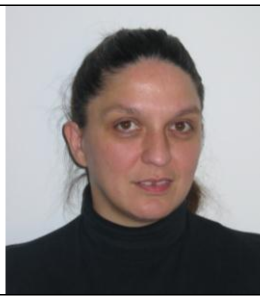
|  |  |  |  |
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**Abstract:** *The problem of determining weights of criteria appears in many contexts in multiple criteria decision making. For solving this task it is necessary to determine individual influence of each considered criterion. Each criterion can be either quantitative - when each of attributes can take different values or qualitative - in which case only ranking of weights of criterias is possible. The weights for each criterion could be provided by experiments, tests or as experts opinions. This paper presents methodology for obtaining an aggregated measure, in the form of arithmetic means, for quantitative decision making. The measure is obtained experimentally by combining statistical methodology - based on regression and factor analysis on one hand and operational research approaches - data envelopment analysis as an objective and analytical hierarchical process as subjective method. For practical demonstration of proposed method authors used data on the students' marks, from the official database at the Faculty of organizational sciences, University of Belgrade, to determine the weights of the first-year subjects in order to predict future students' success on each of the study programs, separately.*

## 1. INTRODUCTION

Experiments are commonly used in the research to test the hypothesis or to validate the results as in the case of choosing an optimum combination of factors in the multifactor or multivariate analysis. Other type of experiments, knows as survey used in the human sciences, has developed in today's information age and is based on data warehousing and data mining on existing databases.

Considering the influence of various factors on the results of the analysis, the choice of the appropriate factors or combinations of factors that will be included in the analysis is a key step in the setup of the experiment. In practice, a large number of factors or criteria could introduce noise in the choice of a suitable alternative. Therefore, the main task should be to determine the number of relevant factors (criteria) and especially their influence on the alternatives which are evaluated. The main goal of this paper is to specify the number of relevant criteria and corresponding weights. Generally speaking, there are two types of weights: coefficients of importance and trade-off indicators. The main difference lies in the way they determine that whether they are defined as criteria weight or possible compensation between them.

Today, the classification of method for determining the criteria weights is not unique [1]. Namely, the division between these methods is carried out in accordance with the author `s concept and the need to solve some practical problems. The main division is: statistical and algebraic, direct and indirect, holistic and decomposed, compensation and non compensation methods.

Two important groups of methodology for determining of the weight of criteria are proposed in this paper:

- standard methods of statistical analysis, including the most commonly used regression and factor analysis, which require a statistically relevant sample and fulfill a strong relationship between the number of member of the observing sample and the number of criteria,
- operational research methods, particularly the multiple criteria analysis methods for determining the weights of criteria to evaluate the objective functions and choosing the optimal number of alternatives and a optimal solution.

In practice, it is often necessary to determine the impact of each factor by decision maker, after implementation of the multi factorial or multiple criteria analysis, in order to evaluate the alternatives in accordance with the purpose of study. Based on the nature of the criteria, which can be quantitative as statistical data and qualitative evaluations, approach to decision making is specified. Qualitative approach only allows selection of the 'best' alternative or determining their ranking with respect to the criteria importance. In many cases it is necessary to determine to what extent the observed alternatives differ, which can be difficult if a large number of factors are included in the analysis. This problem can be solved by applying quantitative methods to determine the influence of individual activities.

All quantitative approaches are based on the matrix

$$\mathbf{M=||}\boldsymbol{m_{ij}}\mathbf{||\ (}\boldsymbol{i}\mathbf{=1,...,}\boldsymbol{r}\mathbf{;\ }\boldsymbol{j}\mathbf{=1,...,}\boldsymbol{s}\mathbf{)} \qquad (1)$$

of the criteria importance $M_1,...,M_r$, characterizing the considered alternatives $A_1,...,A_s$.

The above values may be combined into an aggregate measure if they are dimensionless or normalized. One of the possible normalization procedures is:

$$\boldsymbol{m_{ij}'=m_{ij}\ /\Sigma^{s}_{j=1}\ m_{ij}\ ,\ i=1,...,r} \qquad (2)$$

where $m_{ij}$ 'means the normalized values of the criteria.
The influence of the particular criteria $M_i$ ($i$=1,..., $r$) on the final decision differs, therefore quantitative approaches to decision making requires determining the weights of the criteria $w_i$.

The evaluation of the criteria weights $w_i$ may be, as authors proposed, by statistical or the operational research methods. These methods can be divided in two basic groups: subjective, which depends on the influence of decision maker on the criteria weighting, and objective which are based on application of objective mathematical apparatus on data in decision matrix [1],[2],[3],[4].
In any case, the weights should be normalized, i.e. the total value for all the criteria should be equal to one:

$$\sum_{i=1}^{r} w_i \ \mathbf{=1} \qquad (3)$$

This paper considers the problem of determining the weights of the criteria using statistical weighting with regression and factor analysis from one hand and the methods of operational research, concrete - multiple criteria decision making field, one objective - DEA and one subjective – AHP, from the other hand. These methods are used for determining the relevant subjects for study program selection on Faculty of organization sciences, University of Belgrade.

The paper consists of five sections, followed by references. After the introduction, follows by survey of weights assessment methods, the section of theoretical basis with two subsections is given. The first subsection with statistical methods is divided in two parts - linear regression and factor analysis and the second subsection of multiple criteria methods is also divided in two parts - DEA and AHP. The case study of determining of the relevant subjects for study program selection on Faculty of organization sciences, University of Belgrade based on the first-year marks is presented in the fourth section. The fifth section gives the conclusions where the simply aggregated measure and the influence of the applied methods on proposed aggregation are considered. In this section further possible research directions are also given.

## 2. SURVEY OF WEIGHTS ASSESSMENT METHODS

The basic task which we consider in this paper is to determine a number of relevant criteria and their significance for an object being evaluated in multiple criteria decision making process. The significance of each of evaluated alternatives may vary to a great extent, if it is not clearly defined and it can complicate rating the alternatives.

Each of the mentioned methods for the determination weights of criteria has advantages and disadvantages. Because of that there is a need for a combination of several types of methods of determining the weights.

Authors have made an attempt to make a selection of relevant criteria based on the measure, calculate as arithmetic mean the results obtained by the following methods:

- classical statistical methods of regression and factor analysis and
- two multiple criteria decision methods, one objective Data envelopment analysis (DEA) and one subjective Analytical Hierarchical Process (AHP).

Proposed aggregation is in practice valid for more number of methods in mentioned groups and gives possibility for avoiding of misjudgment of the importance of the criteria that do not correspond to practical experience.

In literature this problem is solved in different ways. The results of one subjective (AHP) and one objective (Entropy) method are aggregated in order to determine criteria weights in the case of performance evaluation of the University departments [5]. Also one good example of integration of DEA and multi-criteria method TOPSIS is given in [6]. The authors developed DEA-based optimization models to facilitate identifying parameter information regarding criterion weights and quantifying qualitative criteria in TOPSIS. The weights determination model that aggregates subjective information provided by the AHP and objective information to form DEA is given in [7].

The approach based on DEA for user-oriented ranking that is developed to help students select an appropriate college is shown in [8]. Multi - criteria AHP method is used in higher education for selection of candidates for teaching positions [9]. AHP method was applied to determine the complicity and priority of selection criteria. The selection of doctoral studies, depending on the objectives that the applicant wants to achieve in his/her career, was done using AHP [10]. Perspectives are set as a pseudo-level in the hierarchy and for each of them the ranking of doctoral studies is done, thus a student can choose based on their preferences. In the paper [11] authors have created the system of knowledge management that determines the minimum distance between existing and new cases and gives a recommendation to a student.

In the paper [12], comparative analyses of DEA and AHP and their aggregation are carried out, according to the importance of the first-year study subjects for choosing an appropriate study program on the example of FOS (Faculty of Organizational Sciences). The importance (weights) of subjects is used for ranking study programs for a particular student, based on his/her success achieved in the subjects that are common for all first- year programs.

The statistical regression methods and analysis of variance (ANOVA) are applied for predicting students results based on their success in high school graduation or entrance exam ([13] and [14]). Also, statistical methods are used to determine the parameters for the assessment of whether the student will withdraw or continue his/her study based on the first- year's results [15]. Also very interesting are the paper [16] and [17] in which are respectively given "An examination of the impact of first-year seminars on correlates of college student retention" and "Predicting student success by mining enrolment data".

From the theory of factor analysis is known (see [18]) that the value of Kaiser-Meyer-Olkin (KMO) measure which is usually calculated for the whole correlation matrix to indicate possibility for factor analysis application, can be calculated for each variable in analysis and in this way, it is possible to examine the suitability of each variable in the analysis.

## 3. THEORETICAL BACKGROUND

### 3.1 Statistical approaches of weights determination

The statistical methods which are objective methods under generally classification same as the objective methods from this classification are basically statistic, are attempt to find the weights from intrinsic information of each evaluation criterion.

Estimation of variance, is measure of dispersion of regression model which is determined with ANOVA from which calculated, for example, F parameters can represent weights of independent variables i.e. criteria.

Also, one of the prerequisites for conducting factor analysis is the relationship between the original variables, and the correlation matrix which enables to detect groups of related variables using for example mentioned Kaiser-Mayer-Olkin statistic i.e. KMO parameter (see [18]).

We repeat, as we said in introduction, that the use of statistic methods of regression and factor analysis requires obvious statistically relevant sample and fulfillment of required relation between the number of elements present in considered sample and the number of considered criteria.

**Regression analysis**

Regression analysis is method for examining the influence more different independent variables for example these p each in n observation i.e. $x_{1i}, x_{2i}, x_{3i}, \dots, x_{pi}$ on one dependent variable for example y with the aim to determine analytic form this connection i.e. model which will be used in analytical and predictive applications. It has deterministic model in which for each value of independent variables exists exactly one value of dependent variable and which can be given in form

$$\mathbf{y=a+ b_1x_1+ b_2x_2+ b_3x_3+\dots+ b_px_p.} \quad (4)$$

where $b_i$ i=1,2,…,p are partial coefficients of regression. In the case when we have non-explained variation of dependent variable in the form of error which we notate $e_i$ as a consequence of influence of random action or non including all existing independent variables we can present multiple regression in the form

$$\mathbf{y=a + b_1x_1+ b_2x_2+ b_3x_3+\dots+ b_px_p.+e_i\,.} \quad (5)$$

The calculation of parameters $a, b_1, b_2, b_3, \dots, b_p$ can be executed with the method of least squares with minimization of sum of squares of the residuals, and in general case notated as:

$$\sum_{i=1}^{n}(y_i - a - b_1x_{1i} - b_2x_{2i} - \cdots - b_px_{pi})^2. \quad (6)$$

In practice, algebraic algorithm for solving arising system of equation is rarely in use than known Gaussian method of multiplication and the most used in empiric researches is ANOVA.

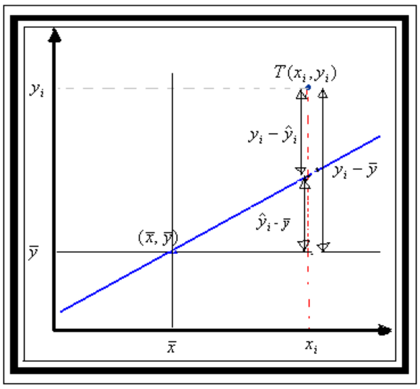


Fig. 1. Equation of regression analysis

Empiric value exempt ion from average could be split to explained exemption (exemption suitable regression value from average) and residual exemption (see Figure 1.):

$$(y_i - \bar{y}) = (\hat{y}_i - \bar{y}) + (y_i - \hat{y}_i) \quad i = 1,2,\dots,n \quad (7)$$

Having in mind that sum of exemptions of all dependent variable individual values is always equal to zero:

$$\sum_{i=0}^{n}(y_i - \bar{y}) = 0 \quad (8)$$

sum of squares of exemption:

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 + \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \quad (9)$$

Obtained equation is known as equation of analysis variance with the following components:

1. corrected sum of squares:

$$ST = \sum_{i=1}^{n}(y_i - \bar{y})^2 \quad (10)$$

2. the sum of regression, which is based on linear regression model, known as explained sum of squares, is the sum of squares of regression value exemptions from average:

$$SP = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 \quad (11)$$

3. Residual, unexplained sum of squares, known as sum of squared errors:

$$SR = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \quad (12)$$

This equation is the base of the analysis of variance of experiment results. Estimation of variance, is measure of dispersion of regression model which is determined with ANOVA from which calculated F parameters basic represented weights of independent variables i.e. criteria what is authors proposal.

**Factor analysis**

Factor analysis groups together attributes that are collinear to form a new factor or attribute, capable of capturing as much of common information of those attributes as possible. Each factor reveals the set of attributes having the highest association with it. The idea under this approach is to account for the highest possible variation in the indicators set using the smallest possible number of factors. Therefore, the factor no longer depends upon the dimensionality of the dataset but it is rather based on the "statistical" dimensions of the data. According to this method, weighting only intervenes to correct for the overlapping information of two or more correlated attributes, and it is not a measure of importance of the associated attribute. Factor analysis provides more options for determining the weights of attributes, i.e. variables in factor analysis. In this paper authors consider one possibility which occurs during designing the correlation matrix. Correlation matrix, which is the basis for the factor analysis implementation, contains simple linear correlation coefficients for each pair of variables. One of the prerequisites for conducting factor analysis is the relationship between the original variables, and the correlation matrix enables to detect groups of related variables.

The Kaiser-Mayer-Olkin statistic, which the KMO measure of the sample adequacy compares the values of the observed correlation coefficients and partial correlation coefficients. Low values of these statistics indicate that the correlation between pairs of the original variables cannot be explained by other variables and there is little justification for the factor analysis application.

Besides the possibility that the value of KMO measure can be calculated for the whole matrix, this measure can be calculated for each variable in analysis. In this way, it is possible to examine the suitability of each variable in the analysis so variables that do not have value of KMO measure which is high enough, may be excluded. This possibility increases the value of KMO measure of the whole matrix (see [18],[19]). Value of KMO measure for each of the attributes (variables) in the factor analysis can be found at the diagonal in the Anti-image matrix. Value of KMO measure can range in the closed interval from 0 to 1. A KMO value 0.9 is the best, while a KMO value bellow 0.5 is unacceptable. The value (1- KMO) in this paper will serve as a basis for determining the weights of each variable in the analysis.

### 3.2 Operational research approaches of criteria weighting

The approaches of determining criteria weights in the field of multiple criteria decision making can be divided into subjective and objective.

Subjective approaches are based on determining criteria weights on the basis of information received by decision makers or experts.

Objective approaches are based on determining criteria weights on the basis of information contained in the decision matrix of some mathematical models.

**Subjective weights determination methods**

Several types of subjective weight restrict methods (such as AHP, Delphi, Swing and multiple criteria decision making) are currently used. These methods are characterized by the subjective setting of weights in the evaluation index, by experts, based on their own experience. Different scholars may give different weights and thus, subjectivity is the major drawback. Remedial measures such as increasing the numbers of experts, properly selecting experts, and so on, can diminish this drawback; however, subjectivity remains. The advantage of the subjective weight restrict method is that experts can reasonably identify the weight index that corresponds to the actual problems. Thus, despite the different placement of weights on the index, the method can still determine the order of priority and avoid conflicts between the reality and the index weights, as can occur in the objective weight restrict method. This study uses AHP.

While the concept of pairwise comparison has been studied by psychologists for over a hundred years the application of the technique, in the context of AHP was promoted in the past twenty years ( see Satty, [20]). Satty's system of nine points has been criticized for producing inconsistent matrix of ratios. To overcome this deficiency and have similar ratios of the consecutive points on the scale Pöyhönen and Hämäläinen [21] have utilized a so called "balanced" scale in their massive experiment with 407 subjects, and Lootsma [22] has suggested a geometric progression scale with progression factor of 2.

The strongest function of AHP is to simplify a complicated system into a hierarchy of processes, each including simple but essential elements. In short, the procedure affects the incentives of each decision making point and the pairwise comparisons between the nominal scales. After the process of quantification, a comparison matrix is established to obtain the eigenvalues vector, representing the weight of each hierarchy, and the eigenvalue. From the above, the corresponding strength and weakness of the individual pairwise comparison used as information for decision-making.

A comprehensive overview of the AHP method and its application in different areas is given in [23].

**Objective approach to determining criteria weights**

In the objective approach to determining criteria weights, criteria are viewed as sources of information and the relative importance of criteria reflects the amount of information contained in any of them. This study uses DEA for weights determination..

DEA is increasingly popular non-parametric method for relative efficiency evaluation" [24],[25] and [26]. DEA method is a set of many approaches and techniques, whose essence is as follows: for each observed Decision Making Unit (DMU) is formed a linear optimization model, whose solution allows assessment of the relative efficiency of the observed units, as well as its comparison with other units. The results of DEA method are information that is multiple useful for managing problems in different DMUs. First of all, it is possible to find out which DMUs are effective and which not. Optimization procedure using the DEA method means the maximum possible level of outputs with minimal involvement or spending of inputs.

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## 4. CRITERIA WEIGHTINGS

There are four study programs at the Faculty of Organizational Sciences (FOS): Information systems (IT), Management (ME), Quality management (KV), and Operations management (OM). On completion of the first year, which has common subjects for all programs, students choose one of these programs. The aim of this paper is to develop a methodology that could help every student to choose a program that would suit him/her best. If a student fails to enroll the suitable program, the order in which the remaining study programs should be selected is determined.

Phases in the implementation of proposed methodology are as follows:

I Gathering data on graduates (entry);
II Evaluation of the first year subjects;
III Selection and ranking of programs for a new student.

### 4.1 GATHERING ENTRY DATA

Data collection is the same as in [11]. The data used here were obtained from the Faculty of Organizational Sciences Students' Service database. The data on 847 graduates who enrolled FOS 2002 and 2006 were selected to be analyzed. For each student, we have collected information on 11 subjects' marks: Economics (EC), Mathematics1 (MA1), Management (MN), Fundamentals of Information Communication Technology (OIKT), Sociology or Psychology (SP), Foreign Language 1 (SJ1), Mathematics 2 (MA2), Fundamentals of Organization (OO), Production systems (PS), Introduction to information Systems (UIS), Foreign Language 2 (SJ2). Belgrade University ranking system: 5 to 10, where 5 means fail and 10 means full mark. The Average mark was used as a key performance that indicates students' success. These data were used in all approaches. Descriptive data statistics is given in Table 1.

Table 1 Descriptive statistics of entry data

| | | | Subjects | | | | | | | | | | | Average | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Program | No. of students | | EK | MA1 | MN | OIKT | SP | SJ1 | MA2 | OO | PS | UIS | SJ2 | Marks in first year | Marks in the end |
| IT | 322 | Average | 7.2 | 8.02 | 7.75 | 8.29 | 8.02 | 7.5 | 8.09 | 8.48 | 8.48 | 8.13 | 7.66 | 7.97 | 8.36 |
| MN | 350 | Average | 7.12 | 7.43 | 7.79 | 7.75 | 8.09 | 7.51 | 7.49 | 8.4 | 8.06 | 7.72 | 7.59 | 7.72 | 8.35 |
| KV | 144 | Average | 6.92 | 7.27 | 7.44 | 7.68 | 7.53 | 7.11 | 7.39 | 8.03 | 7.92 | 7.63 | 7.16 | 7.46 | 8.12 |
| OM | 31 | Average | 7.29 | 7.84 | 7.87 | 8.03 | 8.23 | 7.13 | 8.16 | 8.13 | 8.55 | 7.74 | 7.26 | 7.84 | 8.57 |

### 4.2 SUBJECTS EVALUATION

In determining the priority of the first year subjects that are relevant for predicting future students' success, one objective (DEA) and one subjective (AHP) method from the group of operational research methods and then methods of linear regression and factor analysis are used.

#### Evaluation by DEA

Since the aim of the analysis in this paper is to determine the weight of a subject, the input- oriented DEA model (2-5) is selected. Results are obtained by using specialized software DEA - solver [26], divided into study programs according to students' affiliation.

The relative efficiency is calculated as the ratio of weighted sum of outputs and the weighted sum of inputs. This analysis is not focused on determining the efficiency of the student, as is usually the case when applying DEA, but on the weights. One of the DEA output is a matrix of weights for the inputs (outputs). As the ultimate goal of the research is evaluation of subjects which are inputs of DEA, we shall look at the matrix of inputs weights $v_{ij}, i=1,...,11,\ j=1,...,n_k$ with dimensions $11\times n_{k,}$ where $n_k$ presents the number of students in observed study program ($k$=1,...,4). Based on weights, for every student we calculate average input weights for each study program as follows:

$$z_{ik} = \sum_{j=1}^{n_k} v_{ij}/n_k,\ i=1,...,11,\ k=1,...,4 \qquad (13)$$

Their normalized values are given in the table 2.

#### Evaluation by AHP

AHP is used for attribute weightings (determining the weight of subject) of the first- year study subjects, for each study programs. The hierarchy of problem in is set on two levels with a third, pseudo level, similar to [10]. The described main goal is on the first hierarchy level. As all subjects have different importance (weights) for different study programs ($S_k$, $k$=1,…,4), the main goal will be observed from different perspectives, that is, for each study program separately. As shown in Figure 2, the study programs are set as pseudo levels below the main goal.

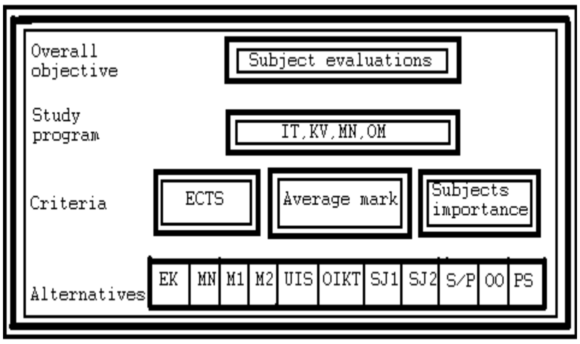


Fig. 2. Subject weighting hierarchy

The importance of subjects for each of the study program is determined on the third level criteria: two objective and one subjective criterion ($C_l$, $l$=1,2,3). The objective criteria are ECTS credits (ECTS) and the average mark. The value of ECTS credits is official information specified in the accreditation program. The values are taken from the Faculty's Student Service database. The subjective criterion is an expert evaluation of relevant subject importance for each study program (Subject importance). It is obvious that subjects are evaluated as set alternatives ($A_i$, $i$=1,…,11), for each study program separately.

In the first stage of evaluation, the comparisons matrix (pair-wise comparison matrix) is created for each study program, based on a subjective expert assessment. In the second phase we enter grades for each alternative, per each criterion. The final result of evaluation is weights of all alternatives (subjects) for each study program separately in relation to the set goal. Weights, which represent the synthesized importance of subjects in choosing study program process ($w_{ik}$, $i$=1,...,11, $k$ = 1, ..., 4) are shown in Table 2.

**Evaluation by regression analysis**

Results are obtained by using the software package SPSS 14.0.0. for regression analysis and univariate ANOVA for each of the study program. In this paper results are presented only for study program IT - information technologies in Figure 3.

Using the normalization of $F$ parameters, relative weights of all criteria is calculated, according to demands of multi-criteria models as it is already described in [12]. Results for this and other three study program are presented in Table 2.

Tests of Between-Subjects Effects

Dependent Variable: AVERAGE

| Source | Type III Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|
| Corrected Model | 101.255[a] | 11 | 9.205 | 102.556 | .000 |
| Intercept | 14.994 | 1 | 14.994 | 167.049 | .000 |
| EK | 1.072 | 1 | 1.072 | 11.942 | .001 |
| MA1 | 2.098 | 1 | 2.098 | 23.370 | .000 |
| MN | .757 | 1 | .757 | 8.430 | .004 |
| OIKT | .209 | 1 | .209 | 2.331 | .128 |
| SP | 2.360 | 1 | 2.360 | 26.289 | .000 |
| SJ1 | .629 | 1 | .629 | 7.009 | .009 |
| MA2 | .753 | 1 | .753 | 8.392 | .004 |
| OO | 1.046 | 1 | 1.046 | 11.654 | .001 |
| PS | 3.676 | 1 | 3.676 | 40.951 | .000 |
| UIS | .711 | 1 | .711 | 7.917 | .005 |
| SJ2 | .236 | 1 | .236 | 2.632 | .106 |
| Error | 27.824 | 310 | .090 | | |
| Total | 22637.711 | 322 | | | |
| Corrected Total | 129.080 | 321 | | | |

**Evaluation by factor analysis**

Using factor analysis, results are obtained with the software package SPSS 14.0.0. One part of SPSS 14.0.0. named "dimension reduction" – "factor" in which we made factor analysis for each of the study program. Figure 4. Shows the determination of KMO parameter for all matrix in the case OM – operations management. From this figure we conclude that for this study program factor analysis is not applicable because in this case KMO parameter has value les than 0.5.

Using the normalization of suitable parameters which are equal to (1-(*KMO*)), it is possible to calculate relative weights of all criteria according to demands of multi-criteria models as it is already described in previous section. Results for this and other three study program are presented in Table 2.

Fig. 3. Univariate ANOVA-IT with SPSS 14.0.0.

KMO and Bartlett's Test

| | | |
|---|---|---|
| Kaiser-Meyer-Olkin Measure of Sampling Adequacy. | | .461 |
| Bartlett's Test of Sphericity | Approx. Chi-Square | 125.901 |
| | Df | 66 |
| | Sig. | .000 |

Fig. 4. KMO measure-OM with SPSS 14.0.0.

**DEA-AHP-linear regression-factor analysis measure**

Flexibility in assigning weights when solving DEA model, as an objective approach, can lead to wrong estimates of individual units (students) under consideration. This is the result of neglecting certain inputs (outputs) that have too large (or too small) values in comparison with other units. In addition standard deviations of subject marks are different what leads to the situation that DEA cannot make discrimination for subjects with a low standard deviation and that it assigns them very low weights.

When determining the importance of alternatives, AHP does not take into account the information about individual marks, but only the average values and subjective values of decision makers. For example, Zeleny [27] suggests that attempt to extract information regarding preferences through questioning the decision maker is a defective process.

ANOVA approach is based on the principle that the criteria having the least variation over the range of alternatives plays minimal role in decision making. This approach is based on the "contrast intensity". It is necessary to remark that the application of this method in the case of OM-operations management is not valid for use because the sample for this study is too small.

At the end, when KMO measure is applied, criteria which are in conflict with larger number of other criteria have a significant role. This approach is based on the "conflicting character" of the evaluated criteria. It is important to notice that the value of KMO for all matrix measure determine applicability this methodology.

Since the importance of entry (subjects) is crucial in this research, it is necessary to combine objective and subjective assessments of operational research methodology with classic statistic assessment using regression and factor analysis. On the one hand, AHP corrects the impact of student marks in some subjects and includes a subjective evaluation of a decision maker and the importance of the subject from the ECTS perspective. On the other hand, the DEA grade brings about objectivity obtained solely only on the basis of empirical values. Also regression analysis is focused on measures of contrast or variability and, on the other hand, the factor analysis is focused on measure of conflicting character of the evaluated criteria.

The objective weights $z_{ik}$, determined by DEA and subjective weight $w_{ik}$ determined by AHP, are combined with the weights $f_{ik}$ determined by regression analysis trough ANOVA- F parameter and the weights $kmo_{ik}$ determined into an aggregated weight as arithmetic mean:

$$agr_{ik} = (z_{ik} + w_{ik} + f_{ik} + kmo_{ik})/4, \; i = 1,...,11 \; k = 1,...,4 \quad (14)$$

Their values are also shown in Table 2.

In literature we can find the aggregated measure in the form of normalized product of two measures, separately in both groups of methods considered in this paper. In operational research approach the objective measure of DEA and subjective AHP are combined for example in [12] and in [27], [28].

Table 2. Weighs obtained by DEA, AHP, regression and factor analysis and aggregated DEA-AHP-regression-factor analysis measure

| Study program | Method | Subjects | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | EK | MA1 | MN | OIKT | SP | SJ1 | MA2 | OO | PS | UIS | SJ2 |
| IT | DEA | 0.116 | 0.239 | 0.089 | 0.093 | 0.126 | 0.076 | 0.034 | 0.104 | 0.105 | 0.001 | 0.017 |
| | AHP | 0.071 | 0.109 | 0.085 | 0.125 | 0.075 | 0.038 | 0.111 | 0.104 | 0.103 | 0.129 | 0.050 |
| | F(ANOVA) | 0.079 | 0.155 | 0.056 | 0.015 | 0.174 | 0.046 | 0.056 | 0.077 | 0.271 | 0.052 | 0.017 |
| | 1-KMO(KEISER-MEYER-OLKIN) | 0.010 | 0.103 | 0.031 | 0.117 | 0.050 | 0.221 | 0.099 | 0.030 | 0.047 | 0.117 | 0.176 |
| | DEA+AHP+F(ANOVA)+(1KMO)/4 | 0.069 | 0.152 | 0.065 | 0.088 | 0.106 | 0.095 | 0.075 | 0.079 | 0.132 | 0.075 | 0.065 |
| MN | DEA | 0.118 | 0.051 | 0.192 | 0.062 | 0.235 | 0.062 | 0.040 | 0.129 | 0.074 | 0.014 | 0.023 |
| | AHP | 0.103 | 0.082 | 0.122 | 0.082 | 0.084 | 0.044 | 0.083 | 0.141 | 0.117 | 0.089 | 0.053 |
| | F(ANOVA) | 0.067 | 0.074 | 0.104 | 0.032 | 0.019 | 0.023 | 0.111 | 0.147 | 0.165 | 0.181 | 0.077 |
| | 1-K MO(KEISER-MEYER-OLKIN) | 0.028 | 0.084 | 0.050 | 0.102 | 0.056 | 0.207 | 0.088 | 0.043 | 0.051 | 0.100 | 0.190 |
| | DEA+AHP+F(ANOVA)+(1KMO)/4 | 0.079 | 0.073 | 0.117 | 0.070 | 0.099 | 0.084 | 0.081 | 0.115 | 0.102 | 0.096 | 0.086 |
| KV | DEA | 0.090 | 0.142 | 0.163 | 0.047 | 0.116 | 0.060 | 0.057 | 0.213 | 0.109 | 0.000 | 0.003 |
| | AHP | 0.087 | 0.082 | 0.122 | 0.082 | 0.084 | 0.044 | 0.083 | 0.141 | 0.133 | 0.089 | 0.053 |
| | F(ANOVA) | 0.007 | 0.071 | 0.005 | 0.087 | 0.102 | 0.016 | 0.084 | 0.101 | 0.449 | 0.004 | 0.075 |
| | 1-KMO(KEISER-MEYER-OLKIN) | 0.062 | 0.051 | 0.085 | 0.068 | 0.064 | 0.221 | 0.059 | 0.035 | 0.110 | 0.063 | 0.182 |
| | DEA+AHP+F(ANOVA)+(1KMO)/4 | 0.062 | 0.087 | 0.094 | 0.071 | 0.092 | 0.085 | 0.071 | 0.123 | 0.200 | 0.039 | 0.078 |
| OM | DEA | 0.066 | 0.008 | 0.344 | 0.123 | 0.273 | 0.005 | 0.060 | 0.005 | 0.115 | 0.000 | 0.000 |
| | AHP | 0.085 | 0.096 | 0.119 | 0.081 | 0.084 | 0.044 | 0.097 | 0.123 | 0.130 | 0.088 | 0.053 |
| | F(ANOVA) | 0.015 | 0.017 | 0.042 | 0.000 | 0.007 | 0.286 | 0.115 | 0.027 | 0.101 | 0.173 | 0.216 |
| | 1-KMO(KEISER-MEYER-OLKIN) | 0.090 | 0.061 | 0.089 | 0.102 | 0.080 | 0.118 | 0.107 | 0.044 | 0.092 | 0.120 | 0.098 |
| | DEA+AHP+F(ANOVA)+(1KMO)/4 | 0.064 | 0.046 | 0.149 | 0.077 | 0.111 | 0.113 | 0.095 | 0.050 | 0.110 | 0.095 | 0.092 |

### 4.3 STUDY PROGRAM SELECTION

Normalized weights of DEA-AHP-linear regression-factor analysis measure in Table 2 can be used for predicting future students' success, after completion of the first year of study, resulting in the recommendation which study program they should select. Students' success is estimated by aggregate weight.

Implementation steps, see [12]:

1. Collect new student's first- year marks;

2. Evaluate study programs based on the aggregated importance of weights by the method of sum of weights:

$$O_k = \sum_{i=1}^{11} agr_{ik} \bullet subject's\_mark_i,\ k = 1,...,4 \qquad (15)$$

3. Determine which study program would suit the student best: ($\max_k O_k$).

In a selection process two approaches were used:

The subjects, which have sum of aggregate weights greater than the given threshold, are considered relevant. The selection of these subjects is based on the values in Table 2.

The selected subjects, according each study program are shown in Table 3.

Table 3. Selected subjects:

| | |
|---|---|
| IT | MA1, OIKT, OO, PS |
| MN | MN, SP, OO |
| KV | OO, MN, PS |
| OM | MN, SP |

### 5. CONCLUSION

Most students select study programs based on personal preferences related to the subjects the study program includes and career qualification. However, if the study program is to be selected after the first-year of study, we can predict students' success based on the first-year grades evaluation and help them select the most appropriate study program.

In this paper, the prediction of success in each of the four study program is based on the weights assigned to the first- year subjects. First, we have estimated 847 students efficiency using DEA. Subject weights were calculated as the average weight assigned to each subject. Second, using AHP the subjects were evaluated as alternatives based on three criteria: ECTS, average mark, and an expert evaluation of a subject importance to different study programs. Subject weights were identified as synthesized weight alternatives. Then using F parameter and ANOVA we have determined the criteria weights estimating theirs contrast of variability and as last estimation of criteria weights we have used their conflicting character with the Kaiser-Meyer-Olkin measure from factor analysis.

Finally, we got weights obtained by aggregate measure which combines all of four mentioned methods in the form of theirs arithmetic mean.

Practically we propose method which is based on aggregation of the methods from basic two groups of criteria weightings, methods of operational research i.e. multiple criteria decision and classic statistic methods. Two sub-groups are recognized in each of these groups, subjective and objective and methods based on regression and factor analysis, respectively.

It is evident that aggregation in the form of arithmetic mean is better when we use more different methods from the both considered groups.

In literature and practice, there are several methods which combines results of subjective and objective operational research approach, but that is not the case for the methods of regression and factor analysis [29].

In the future, authors will consider the other potential measures of variability, for example standard deviation. Also, authors will consider, potential measures of character conflicting of the evaluated criteria, for example squared factor loadings and weights from factor analysis method.

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