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Outage Analysis of Partial Relaying Selection System in Interference-limited Environment

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***Abstract* - The outage probability of dual-hop links with partial relay selection over Rayleigh fading channel in the presence of interferences is analysed. The outage expressions for amplify-and-forward (AF) and decode-and-forward (DF) relaying with relay selection, based on the instantaneous signal-to-interference ratio (SIR), are derived. Numerical results for both types of relays are presented and compared.**

***Key words:* Amplify-and-forward relays, decode-and-forward relay, interference, outage probability, partial relay selection**

I. INTRODUCTION

Dual-hop cooperative relaying technology is often used as an efficient solution for improving the performance of wireless networks. The key idea of dual-hop system is that the transmission between the source and the destination is performed via a relay when the direct link is in a deep fade [1-2]. This technology has advantage of extending the coverage and increasing throughput without using large power at the transmitter. Among different relay protocols, the most two commonly used are amplify-and-forward (AF) and decode-and-forward (DF). The AF relays can be classified into two subcategories: the channel state information (CSI)-assisted relays and the fixed-gain relays. The CSI-assisted relays use output power control based on the channel state information from the previous hop, while fixed-gain relays produce variable output power depending on the strength of the incoming signal [1-2].

In wireless systems, cooperative users and relays may experience interferences from adjacent cells using the same frequency and co-channel interferences are one of the major causes of the system performance degradation [3]. Dual-hop system performance in the presence of interferences was investigated in [4-7], but those papers considered the dual-hop of single relay systems. In the systems with multiple relays, one of the ways for further improvement of performance is the use of diversity techniques. One of the simplest techniques is the partial relay selection scheme which was proposed in [8]. These schemes require the CSI either at source-relay links, or at relay-destination links. Selection scheme where the relay node is selected based only on CSI of the first hop is investigated in [9]. The end-to-end outage probability with partial relay selection in the presence of multiple interference only at the relay is given in [10], while the performance of the system with interference-limited destination are presented in [11].

We investigate the outage performance of multiple relay dual-hop system in an interference-limited environment with the partial relay selection scheme using CSI-assisted AF and DF protocol. The considered selection scheme requires the CSI only from the first hop. In the presence of interference, the relay node is selected based on the estimated signal-to-interference ratio (SIR). An exact closed-form expression for the outage probability is derived.

II. SYSTEM AND CHANNEL MODEL

We consider dual-hop system consisting of one source, one destination and L relays R_k ($k = 1, 2,...L$). The relays and destination terminals are corrupted by the multiple co-channel interference and additive white Gaussian noise. However, since we assume the interference-limited environment, the thermal noise may be neglected. The transmission is performed only via relays because of the unavailability of the direct link. The source selects the best source-relay link by monitoring the source-relay links quality. Selection procedure is based on the higest instantaneous SIR. The selected relay R_k amplifies and forwards the received signal to the destination when CSI-assisted AF relays are employed. In the system with DF relays, selected relay decodes and forwards the re-encoded signal to the destination.

Let us assume that the source S transmits the desired signal s_0 having an average power P_S. The signal received at the CSI-assisted relay R_k in interference-limited fading environment with M co-channel interferers can be written as

$$r_{Rk} = \sqrt{P_s} h_{SRk} s_0 + \sum_{p=0}^{M} \sqrt{P_{kp}} h_{kp} s_{kp} \quad (1)$$

where h_{SRk} is the fading amplitude of the channel between S and R_k, h_{kp} is the fading amplitude of the channel from the p-th interfering terminal to relay R_k and s_{kp} denotes the undesired signal with average power P_{kp} from p-th co-channel interference at the input of R_k. The signal r_{Rk} is then multiplied by the gain G at the relay R_k and retransmitted to the destination D which suffers from N co-channel interferers. The received signal at the destination D can be written as

$$r_D = h_{RkD} G r_{Rk} + \sum_{q=0}^{N} \sqrt{P_q}\, g_q s_q \tag{2}$$

where h_{RkD} is the fading amplitude of the channel between R_k and D, g_q is the amplitude of the channel from the q-th interfering terminal to the destination, s_q represents the q-th cochannel undesired signal at D with average power P_q. The overall SIR at the receiving end can be expressed as

$$z_{eq} = \frac{|h_{SRk}|^2 |h_{RkD}|^2 G_k^2 P_s}{|h_{RkD}|^2 G_k^2 \sum_{p=0}^{M} P_{kp} |h_{kp}|^2 + \sum_{q=0}^{N} P_q |g_q|^2}. \tag{3}$$

In [5], the gain of the relay depends only on the CSI from source-relay channel. However, we assume that the relay R_k has available instantaneous CSI from the source-relay including all interference channels. Therefore, the gain G_k is given by [7]

$$G_k^2 = \frac{P_{Rk}}{|h_{SRk}|^2 P_s + \sum_{p=0}^{M} P_{kp} |h_{kp}|^2}. \tag{4}$$

where P_{Rk} is the power of the transmitted signal at the output of the relay R_k.

Therefore, the instantaneous equivalent end-to-end SIRs in this case can be obtained by substituting (4) in (3) and has a form [4]

$$z_{eq} = \frac{\dfrac{|h_{SRk}|^2 P_s}{\sum_{p=0}^{M} P_{kp} |h_{kp}|^2} \dfrac{|h_{RkD}|^2 P_r}{\sum_{q=0}^{N} P_q |g_q|^2}}{\dfrac{|h_{RkD}|^2 P_r}{\sum_{q=0}^{N} P_q |g_q|^2} + \dfrac{|h_{SRk}|^2 P_s}{\sum_{p=0}^{M} P_{kp} |h_{kp}|^2} + 1} = \frac{z_1 z_2}{z_1 + z_2 + 1} \tag{5}$$

where $z_1 = |h_{SRk}|^2 P_s / \sum_{p=1}^{M} P_{kp} |h_{kp}|^2$, $z_2 = |h_{RkD}|^2 P_r / \sum_{q=1}^{N} P_q |g_q|^2$ are the instantaneous SIR in the first and the second hop, respectively.

On the other hand, in the system with DF relays, the overall system performanse is determined by SIR at relay, z_1 and destination, z_2.

When the fading in the first hop has Rayleigh distribution and all M interferers are independent and identically distributed Rayleigh random variables, the instantaneous SIR per hop at k-th relay has the probability density function (pdf) [12]:

$$p_{R_k}(z) = \frac{M}{\bar{z}_{Rk}} \frac{1}{(1 + z/\bar{z}_{Rk})^{M+1}}, \tag{6}$$

where $\bar{z}_{Rk}$ is the ratio of the average signal power of the first hop (S-R_k) and the average single interference power at k-th relay. The cumulative distribution function (cdf) for instantaneous SIR is

$$P_{R_k}(z) = 1 - \frac{1}{(1 + z/\bar{z}_{Rk})^{M}}. \tag{7}$$

The best relay, out the L available relays, is selected based on the highest instantaneous SIR, and the cdf of SIR, z_1 is [13]

$$P_{z_1}(z) = \prod_{k=1}^{L} P_{R_k}(z). \tag{8}$$

Under the assumption of $\bar{z}_{Rk} = \bar{z}_1$ for $k = 1, 2, \ldots L$, using the series representation of binomial formula [14, Eq. (1.111)] we have

$$P_{z_1}(z) = \sum_{p=0}^{L} \binom{L}{p} \frac{(-1)^p}{(1 + z/\bar{z}_1)^{Mp}}. \tag{9}$$

The selected relay forwards the signal transmitted by the source to the destination which is corrupted by N independent and identically distributed co-channel Rayleigh intreferers. The pdf and cdf of instantaneous SIR, z_2 of the second hop are respectively [12]:

$$p_{z_2}(z) = \frac{N}{\bar{z}_2} \frac{1}{(1 + z/\bar{z}_2)^{N+1}}. \tag{10}$$

and

$$P_{z_2}(z) = 1 - \frac{1}{(1 + z/\bar{z}_2)^{N}} \tag{11}$$

where $\bar{z}_2$ is the average second hop (R_k-D) signal-to-single interference power ratio.

III. OUTAGE ANALYSIS

In the interference-limited systems, the outage probability is defined as the probability that the instantaneous equivalent SIR, λ_{eq}, falls below a predetermined threshold, λ_{th} [3].

A. Amplify-and-forward relays

The outage probability of the instantaneous equivalent SIR when CSI-assisted AF relays are implemented can be expressed as [2]:

$$\begin{aligned} P_{eq}^{AF}(\lambda_{th}) &= \Pr(z_{eq} < \lambda_{th}) \\ &= \int_0^\infty P_{z_1}\left(\frac{z_1 z_2}{z_1 + z_2 + 1} \le \lambda_{th} \middle| z_2 \right) p_{z_2}(z_2) \mathrm{d}z_2, \end{aligned} \tag{12}$$

After applying some algebraic manipulations eq. (12) can be expressed as, similar to the [15]

$$P_{eq}^{AF}(\lambda_{th}) = 1 - \int_0^\infty C_{z_1}\left(\lambda_{th} + \frac{\lambda_{th}^2 + \lambda_{th}}{x} \right) p_{z_2}(\lambda_{th} + x) \mathrm{d}x \tag{13}$$

where $C_{z_1}(\cdot)=1-P_{z_1}(\cdot)$ is the complementary cdf of z_1. Therefore, using (9) and (10), the outage probability can be expressed as:

$$P_{eq}^{AF}(\lambda_{th})=1+\frac{N}{\bar{z}_2}\sum_{p=1}^{L}\binom{L}{p}(-1)^p \times\int_0^\infty\left(1+\frac{\lambda_{th}}{\bar{z}_1}+\frac{\lambda_{th}^2+\lambda_{th}}{\bar{z}_1 x}\right)^{-Mp}\left(1+\frac{\lambda_{th}+x}{\bar{z}_2}\right)^{-N-1}\mathrm{d}x. \quad (14)$$

After some mathematical manipulations and using [16, Eq. (07.23.07.0002.01)], we get a novel closed-form expression for the outage probability

$$P_{eq}^{AF}(\lambda_{th})=1+\frac{N\bar{z}_2^N}{(\lambda_{th}^2+\lambda_{th})^N}\sum_{p=1}^{L}\binom{L}{p}(-1)^p \times\frac{\Gamma(N)\Gamma(Mp+1)}{\Gamma(Mp+N+1)}\bar{z}_1^{Mp}(\bar{z}_1+\lambda_{th})^{-Mp+N} \times{}_2F_1\left(N+1,N,Mp+N+1,1-\frac{(\bar{z}_1+\lambda_{th})(\bar{z}_2+\lambda_{th})}{\lambda_{th}^2+\lambda_{th}}\right), \quad (15)$$

where ${}_2F_1(a,b,c,z)$ is Gauss hypergeometric function [14, eq. (9.100)]. By setting $L = M = N = 1$ (15) can be simplified to the result that has been found in [4, Eq. (16)]. Also, for L=1, (15) can be simplified to the published result in [7, eq. (16)].

D. Decode-and-forward relays

The outage probability of dual-hop DF rely system can be defined as the probability that either one of the two hops is in outage, i.e. when instantaneous SIR of any hop is below a predetermined outage threshold

$$P_{eq}^{DF}(\lambda_{th})=1-(1-P_{z_1}(\lambda_{th}))(1-P_{z_2}(\lambda_{th})). \quad (16)$$

where $P_{z_1}(\lambda_{th})$ and $P_{z_2}(\lambda_{th})$ are given by (8) and (11), respectively.

IV. NUMERICAL RESULTS

Numerical results for outage probability, obtained according to (15) for AF system and (16) for DF relaying system, are shown in this Section.

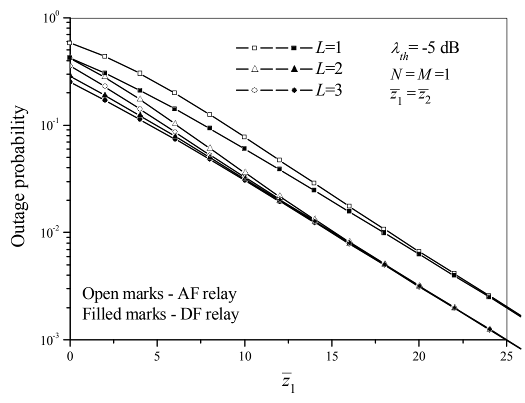

Fig. 1. Outage probability as a function of the first hop average SIR for different number of relays

Fig. 1 shows the outage performance as a function of the first hop average SIR $(\bar{z}_1)$ for different number of relays in the presence of single dominant interference at relays and destination. The systems with DF relays outperform those with AF relays, especially for low values of the average SIR. The highest performance gain is obtained if two relays with partial selection are used instead of one relay. In the mid-high average SIR range the increase of the number and type of relays has no influence on the system performance.

Fig. 2. shows the outage performance for different number of interferences at relays (L=2) and destination and for $\bar{z}_2$=20 dB. As the number of interference increases, the outage probability also increases. The largest performance degradation is present when the number of interferences increases from one to two. For low average SIR values system with DF relays outperforms system with AF relays. The type of relay has no influence in the system performance for higher values of the average signal-to-single interference power ratio.

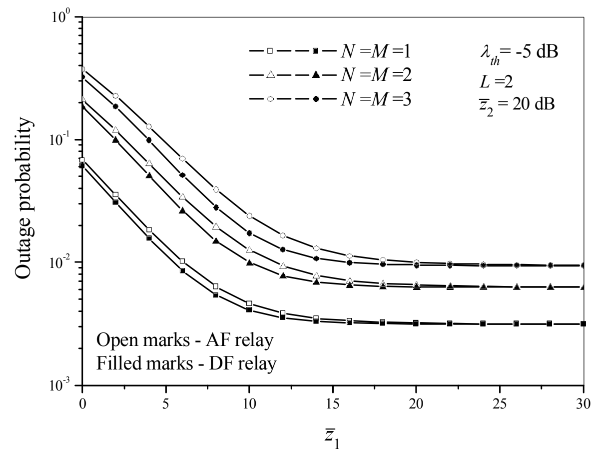

Fig. 2. Outage probability as a function of the first hop average SIR for different number of interference

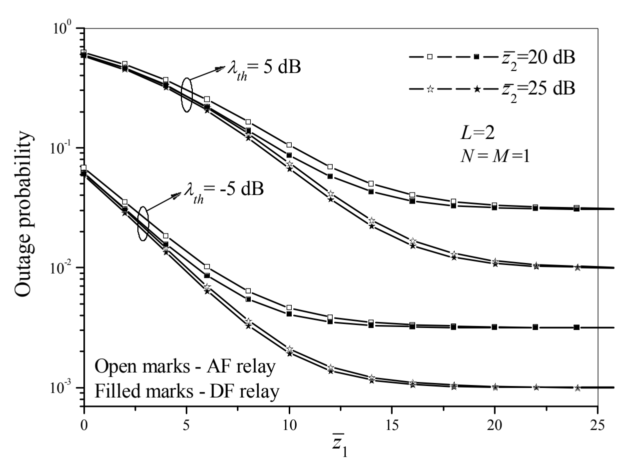

Fig. 3. Outage probability as a function of the first hop average SIR for different values of outage threshold

The outage probability for different values of threshold and for different values of second hop average SIR, is shown in Fig. 3. The outage probability increases with the increase of the outage threshold. With increasing second hop average SIR, the system performance is getting better. One can notice that for low values of the first hop average SIR there is improvement in system performance when DF relays are used instead of AF relays.

V. CONCLUSION

Performance of dual-hop system with partial relays selection under assumption that relays and destination are affected by multiple co-channel interferences has been studied. We have derived expressions for the outage probability of the instantaneous SIR for CSI-assisted amplify-and-forward and decode-and-forward relays operating over Rayleigh fading channels. The effects of number and type of relays, outage threshold, number of co-channel interferences, on the system's performance, were interpreted. Proposed analysis can be used in design of a cellular mobile system to determine optimal values of system parameters to provide a reasonable indication of signal outage.

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