EXACT BIT ERROR PROBABILITY EXPRESSION FOR QAM OVER GAMMA SHADOWED NAKAGAMI-M FADING CHANNEL

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Abstract – *In this paper, we derive the new closed-form expressions for bit error rate (BER) in detecting quadrature amplitude modulation signals transmitted over gamma-shadowed Nakagami-m fading channels. By using those expressions, the effects of fading and shadowing severity and average signal-to-noise ratio on BER performance are analyzed.*

1. INTRODUCTION

Quadrature amplitude modulation (QAM) is widely used technique to achieve high transmission rate without increasing the bandwidth [1], [2]. Calculation of bit error rate (BER) occupied attention of researches, first in the channel with additive white Gaussian noise (AWGN), and after that, in the channels with different types of fading [3]-[6].

In wireless communication, fading is one of the main problems. Several statistical models describe fading, e.g. Rayleigh, Rice and Nakagami model. Nakagami model is more general than Rayleigh and Rice, and therefore it is very used in observations. The basis in all these fading models is the assumption that the average signal power is constant. But, the presence of tall structures in the path of the signal and the existence of multiple scattering may lead to the case where the received average power becomes random. This phenomenon is called shadowing. Firstly, the shadowing was modeled by lognormal distribution, but the gamma distribution has been accepted as more convenient. Since fading (short term) and shadowing (long term fading) occur simultaneously in wireless systems, it is necessary to have models that can describe the faded and shadowed channel [7]-[10]. In this paper, we considered composite signal described by gamma-shadowed Nakagami-*m* fading model.

The analyses BER performances of a two-dimensional amplitude modulation, *M*-ary square QAM and an *I*x*J* rectangular QAM signals over AWGN channel was presented in [3]. The same BER performances over the Nakagami-*m* channel were shown in [4]. In this paper, the new closed-form expressions for BER in detecting QAM signal transmitted over gamma-shadowed Nakagami-*m* fading channel are derived. The BER dependence on fading and shadowing severity and average signal-to–noise ratio (SNR) per bit is presented.

2. SYSTEM MODEL

Using the *M*-ary square QAM modulation, transmitted signal consists of two independently amplitude-modulated carriers in quadrature expressed by [1], [2]

$$s(t) = A_I \cos(2\pi f_c t) - A_j \sin(2\pi f_c t),\ 0 \le t \le T, \qquad (1)$$

where A_I and A_J are the amplitudes of in-phase and quadrature components, f_c is the carrier frequency, and T is the symbol period. Depending on the number of possible symbols M, two distinct QAM constellations can be distinguish: square constellations with even number of bits per symbol, and rectangular constellations where the number of bits per symbol is odd.

In M-ary square QAM, $\log_2 M$ bits of the serial information stream are mapped on a two-dimensional signal constellation using Gray coding. In (1), A_I and A_J are selected independently over the set $\{\pm d, \pm 3d, ..., \pm(\sqrt{M}-1)d\}$ where $2d$ is the Euclidean distance between two adjacent signal points.

In this paper, we consider the case when the signal is transmitted from the transmitter to the receiver via channel with gamma-shadowed Nakagami-*m* fading.

Let the received signal envelope r has Nakagami distribution given by [7]

$$p_{r/\Omega}(r/\Omega) = \frac{2m^m r^{2m-1} e^{-\frac{m}{\Omega}r^2}}{\Gamma(m)\Omega^m},\quad r > 0, \qquad (2)$$

where m $(0.5 \le m < \infty)$ is the Nakagami parameter, Ω is the average power $\Omega = E[r^2]$ with E denoting mathematical expectation and $\Gamma(.)$ is the gamma function. The m parameter refers to the fading severity. In the case m=1, we have Rayleigh fading, and $m=\infty$ is the no-fading case. When $m>1$, the Nakagami-*m* distribution behaves like the Rician distribution.

In the case when the shadowing is present, Ω is random variable and has gamma distribution given by [7]

$$p_\Omega(\Omega) = \frac{m_s^{m_s} \Omega^{m_s - 1} e^{-\frac{m_s}{\Omega_s}\Omega}}{\Gamma(m_s)\Omega_s^{m_s}},\quad \Omega > 0 \qquad (3)$$

where $\Omega_s = E[\Omega]$ is the gamma shadow area mean power. The parameter m_s $(0 < m_s < \infty)$ infleunces on the shadowing severity. In the case $m=\infty$, shadowing is not exist.

The composite envelope r of the gamma-shadowed Nakagami-m faded signal is:

$$p(r) = \int_0^\infty p_{r/\Omega}(r/\Omega) p_\Omega(\Omega) d\Omega \tag{4}$$

Substituting (2) and (3) in (4), we have

$$p_r(r) = \frac{4}{\Gamma(m)\Gamma(m_s)} \left(\frac{mm_s}{\Omega_s}\right)^{\frac{m+m_s}{2}} \times r^{m+m_s-1} K_{m_s-m}\left(2r\sqrt{\frac{mm_s}{\Omega_s}}\right) \tag{5}$$

where $K_v(.)$ is the modified Bessel function of the second kind and order v and $\Omega_s = E[r^2] = \overline{r^2}$ is the average power.

Now, in the presense of gamma-shadowed Nakagami-m fading, the QAM signal is:

$$s_r(t) = r \cdot A_I \cos(2\pi f_c t) - r \cdot A_j \sin(2\pi f_c t), \tag{6}$$

where r is the composite envelope of received signal that has the distribution given by (5).

The instantaneous SNR per symbol, ρ_s, and the average SNR, ρ_{0s}, are related by:

$$\frac{r^2}{\overline{r^2}} = \frac{\rho_s}{\rho_{0s}}, \quad r > 0, \ \rho_s > 0\,. \tag{7}$$

The instantaneous SNR per symbol is $\rho_s = \rho \log_2 M$, where ρ is the instantaneous SNR per bit. The average SNR per symbol is $\rho_{0s} = \rho_0 \log_2 M$, where ρ_0 is the instantaneous SNR per bit.

The distribution of the SNR per bit can be found using standard technique of transforming random variables:

$$p(\rho) = \frac{2}{\Gamma(m)\Gamma(m_s)} \left(\frac{mm_s}{\rho_0}\right)^{\frac{m+m_s}{2}} \times \rho^{\frac{m+m_s-2}{2}} K_{m_s-m}\left(2\sqrt{\frac{mm_s}{\rho_0}\rho}\right). \tag{8}$$

3. GENERAL BER EXPRESSION OF *M*-ARY SQUARE QAM OVER THE GAMMA-SHADOWED NAKAGAMI-*m* FADING CHANNEL

Using [3, eq. (14)], the conditional kth bit error probability of M-ary square QAM can be expressed by

$$P_{b/\rho}(k) = \frac{1}{\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \Bigg\{ (-1)^{\left\lfloor \frac{i\cdot 2^{k-1}}{\sqrt{M}} \right\rfloor} \times \left(2^{k-1} - \left\lfloor \frac{i\cdot 2^{k-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor\right) \times \operatorname{erfc}\left((2i+1)\sqrt{\frac{3\log_2(M)\rho}{2(M-1)}}\right) \Bigg\} \tag{9}$$

where erfc(.) is the complementary error function and ρ denotes the instantaneous SNR per bit. Since the instantaneous SNR per bit is random variable, in order to obtain average the kth bit error probability of M-ary square QAM, it is necessary to average previous expression. So, we have

$$P_b(k) = \int_{\rho=0}^{\infty} P_{b/\rho}(k) p(\rho) d\rho \tag{10}$$

where $p(\rho)$ is given by (9).

Now, using (9) and (10), the bit error probability $P_b(k)$ (kth bit is in error) can be expressed as

$$P_b(k) = \frac{1}{\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \Bigg\{ (-1)^{\left\lfloor \frac{i\cdot 2^{k-1}}{\sqrt{M}} \right\rfloor} \times \left(2^{k-1} - \left\lfloor \frac{i\cdot 2^{k-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor\right) \times \int_{\rho=0}^{\infty} \operatorname{erfc}\left((2i+1)\sqrt{\frac{3\log_2(M)\rho}{2(M-1)}}\right) p(\rho) d\rho \Bigg\}. \tag{11}$$

Integral in (11) can be solved by representing complementary error function and modified Bessel function in terms of Meijer's G functions using [8, eqs. (03.04.26.0009.01) and (06.27.26.0006.01)], and afterwards using [8, eq. (07.34.21.0011.01)]. The eq. (11) becomes

$$P_b(k) = \frac{1}{\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \Bigg\{ (-1)^{\left\lfloor \frac{i\cdot 2^{k-1}}{\sqrt{M}} \right\rfloor}$$

$$\times\left(2^{k-1}-\left\lfloor\frac{i\cdot 2^{k-1}}{\sqrt{M}}+\frac{1}{2}\right\rfloor\right)\frac{1}{\sqrt{\pi}\Gamma(m)\Gamma(m_s)}$$

$$\times G_{3,2}^{2,2}\left(\frac{(2i+1)^2 3\log_2(M)\rho_0}{2(M-1)mm_s}\left|\begin{matrix}1-m_s, & 1-m, & 1\\ 0, & 1/2 & \end{matrix}\right.\right)\Bigg\} \quad (12)$$

where $G_{m,n}^{p,q}(\cdot)$ denotes Meijer G-function.

Using [3, eq. (16)], the exact expression of average BER of M-ary square QAM is given by

$$P_b=\frac{1}{\log_2\sqrt{M}}\sum_{k=1}^{\log_2\sqrt{M}}P_b(k) \quad (13)$$

where $P_b(k)$ is given by (12).

An approximate BER expression for M-ary square QAM can be obtained from (13) by neglecting higher order terms. If only the first and the second terms (i=0,1) in (12) are considered, we have

$$P_b\cong\frac{\sqrt{M}-1}{\sqrt{M}\log_2(\sqrt{M})\Gamma(m)\Gamma(m_s)\sqrt{\pi}}$$

$$\times G_{3,2}^{2,2}\left(\frac{3\log_2(M)\rho_0}{2(M-1)mm_s}\left|\begin{matrix}1-m_s, & 1-m, & 1\\ 0, & 1/2 & \end{matrix}\right.\right)$$

$$+\frac{\sqrt{M}-2}{\sqrt{M}\log_2(\sqrt{M})\Gamma(m)\Gamma(m_s)\sqrt{\pi}}$$

$$\times G_{3,2}^{2,2}\left(\frac{3\log_2(M)\rho_0}{2(M-1)mm_s}\left|\begin{matrix}1-m_s, & 1-m, & 1\\ 0, & 1/2 & \end{matrix}\right.\right). \quad (14)$$

For high SNR, the first term (i=0) is dominant in (13). Then the BER of M-ary square QAM can be approximated to a certain degree of accuracy as

$$P_b\cong\frac{\sqrt{M}-1}{\sqrt{M}\log_2(\sqrt{M})\Gamma(m)\Gamma(m_s)\sqrt{\pi}}$$

$$\times G_{3,2}^{2,2}\left(\frac{3\log_2(M)\rho_0}{2(M-1)mm_s}\left|\begin{matrix}1-m_s, & 1-m, & 1\\ 0, & 1/2 & \end{matrix}\right.\right). \quad (15)$$

4. GENERAL BER EXPRESSION OF *M*-ARY RECTANGULAR QAM OVER THE GAMMA-SHADOWED NAKAGAMI-*m* FADING CHANNEL

In the similar way, using [3, eq. (20), (21), (22)], the average BER of M-ary rectangular QAM can be determined. The average BER of $I\times J$ rectangular QAM over gamma-shadowed Nakagami fading channel is

$$P_b=\frac{1}{\log_2(I\cdot J)}\left(\sum_{k=1}^{\log_2 I}P_I(k)+\sum_{l=1}^{\log_2 J}P_J(l)\right) \quad (16)$$

where

$$P_I(k)=\frac{1}{I}\sum_{i=0}^{(1-2^{-k})I-1}\Bigg\{(-1)^{\left\lfloor\frac{i2^{k-1}}{I}\right\rfloor}$$

$$\times\left(2^{k-1}-\left\lfloor\frac{i2^{k-1}}{I}+\frac{1}{2}\right\rfloor\right)\frac{1}{\sqrt{\pi}\Gamma(m)\Gamma(m_s)}$$

$$\times G_{3,2}^{2,2}\left(\frac{(2i+1)^2 3\log_2(I\cdot J)\rho_0}{(I^2+J^2-2)mm_s}\left|\begin{matrix}1-m_s, & 1-m, & 1\\ 0, & 1/2 & \end{matrix}\right.\right)\Bigg\} \quad (17)$$

and

$$P_J(l)=\frac{1}{J}\sum_{j=0}^{(1-2^{-l})J-1}\Bigg\{(-1)^{\left\lfloor\frac{j2^{l-1}}{J}\right\rfloor}$$

$$\times\left(2^{l-1}-\left\lfloor\frac{j2^{l-1}}{J}+\frac{1}{2}\right\rfloor\right)\frac{1}{\sqrt{\pi}\Gamma(m)\Gamma(m_s)}$$

$$\times G_{3,2}^{2,2}\left(\frac{(2i+1)^2 3\log_2(I\cdot J)\rho_0}{(I^2+J^2-2)mm_s}\left|\begin{matrix}1-m_s, & 1-m, & 1\\ 0, & 1/2 & \end{matrix}\right.\right)\Bigg\}. \quad (18)$$

Note that for $I=J=\sqrt{M}$, (16) reduces to (13), i.e. the average BER of square M-ary QAM is the same as for the rectangular.

For high SNR, the terms with i=0 and j=0 in (17) and (18), respectively, will be dominant, so an approximate BER of M-ary rectangular QAM can be obtained from (16)

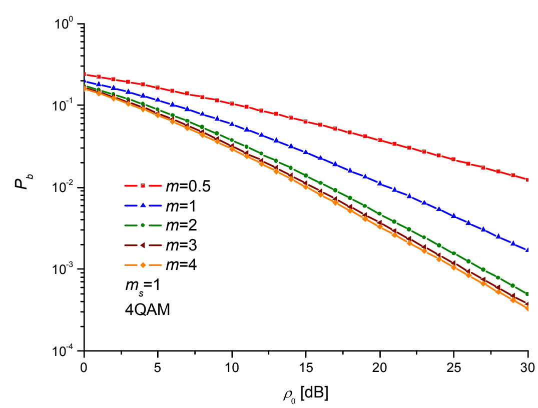

Fig.1. *BER dependence on average SNR for different fading severeness.*

$$P_b = \frac{\left(\frac{I-1}{I} + \frac{J-1}{J}\right)}{\sqrt{\pi}\Gamma(m)\Gamma(m_s)\log_2(I \cdot J)}$$

$$\times G_{3,2}^{2,2}\left(\frac{3\log_2(I \cdot J)\rho_0}{(I^2+J^2-2)mm_s} \middle| \begin{matrix} 1-m_s, & 1-m, & 1 \\ 0, & 1/2 & \end{matrix} \right). \quad (19)$$

When $I = J = \sqrt{M}$, (20) reduces to (15).

5. NUMERICAL RESULTS

The numerical results are obtained by expressions (12) and (13) for square QAM, and (16), (17) and (18) for rectangular QAM.

Fig.1. shows BER dependence on average SNR for different values of fading parameter *m*. We can see that the performance of the system is the worst when the *m*=0.5. With decreasing value of fading parameter *m*, we have severe fading.

Fig.2. shows BER dependence on average SNR for different values of shadowing parameter m_s, while the fading parameter is *m*=1. With increasing values of SNR, the BER is lower. We can see that the performance of the system is the worst when the m_s=0.5. For SNR=25 dB, the value of BER is 0.029 when the value of m_s=0.5, and 0.002 when the value of m_s=4.When the value of the parameter m_s is lower, the system has worse performance, i.e. the influence of shadowing is bigger.

Fig.3. shows how the parameter m_s influence on average SNR per bit during 8QAM. Lower values of m_s require larger power of the signal if the certain BER is wanted, i.e. the shadowing is more expressed. For m_s=2, SNR is 24.97 when the value of BER is 10^{-4}, and SNR is 12.99 when the value of BER is 10^{-2}. When better perfomance system is wanted (lower value of BER), larger power of the signal is needed.

Fig.4. shows the results for different type of QAM. It is noticed that with higher order of QAM, the performance of BER is worse, but the larger amount of information is transmitted.

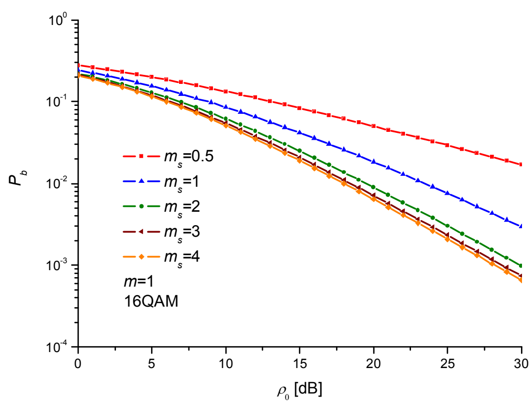

Fig.2. *BER dependence on average SNR for different shadowing severeness.*

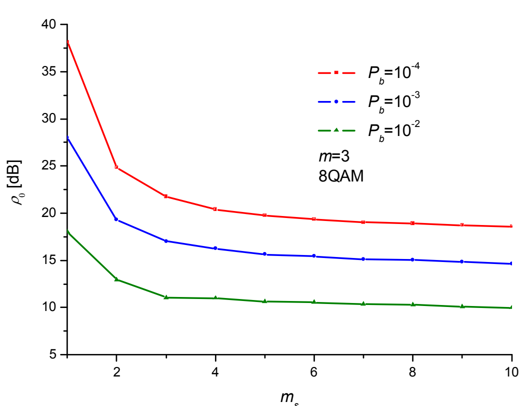

Fig.3. *Average SNR dependence on parameter m_s for different values of BER.*

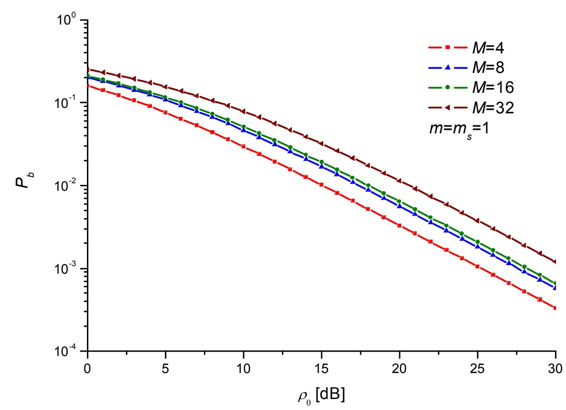

Fig.4 *BER dependence on average SNR in the gamma-shadowed Nakagami-m fading channel for different types of QAM* .

CONCLUSION

In this paper we have analyzed *M*-ary QAM transmission over the channel with gamma-shadowed Nakagami-*m* fading. The closed-form experssions for BER have been derived and used for observing the BER performances. The effects of the fading and shadowing parameters and average signal-to-noise ratio on the BER performances have been noted.

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